Study of Reasoning Rules in Discrete Mathematical Propositional Calculus

Shao Jingjing[1]

Institute of science and technology, Dianchi College of Yunnan University, Kunming, Yunnan, 650228

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Abstract: Mathematical logic is the science of reasoning, especially in mathematics. In the process of logical reasoning, if there are more propositional arguments, it is not convenient to use the column truth table or equivalent calculus, so the method of constructing proof should be introduced. This proof method must be carried out under the given rules, which requires students to have a high degree of abstract thinking ability and rigorous mathematical reasoning skills. The important rules of reasoning are the core of this part. Only by exploring the rules can we master the reasoning system well.

Mathematical logic has two main parts: propositional calculus and predicate calculus. The axiom system of the perpetual truth formula in propositional calculus is a system that gives a number of perpetual truth formulas (or rules of reasoning) and then gives a number of rules of reasoning derived from the perpetual truth formula from which all the perpetual truth formulas are derived. The reasoning theory of proposition should master important reasoning rules, make clear the formal structure of reasoning, master the method to judge whether reasoning is correct and apply P rules, T rules and CP rules [1]. The difficulty of mathematical logic is the reasoning theory and application, which should first find the learning method of the important reasoning rules in propositional calculus.

1. Important rules of propositional calculus

The main function of logic is to provide a simple system of axioms for reasoning. A theory of systems is called a theory of reasoning because it does not involve the derivation of a conclusion from a premise. In the process of logical reasoning, the method of introducing the construction system must be carried out under the given rules and the common rules are based on the eternal-true implication. There are 8 important inference rules [2]. In order to study the methods to have a better understanding of them, a rule[3] is added, namely the following 9 rules:

Rule 1（supplement）: $P \Rightarrow P$

Rule 2（adding-up）: $P \Rightarrow P \vee Q$

Rule 3（reduced）: $P \wedge Q \Rightarrow P$

Rule 4（modus ponens）: $(P \to Q) \wedge P \Rightarrow Q$

Rule 5（modus tollendo ponens）: $(P \to Q) \wedge \neg Q \Rightarrow \neg P$

Rule 6（disjunctive syllogism）: $(P \vee Q) \wedge \neg P \Rightarrow Q$

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Rule 7（syllogism of premises）: $(P \to Q) \wedge (Q \to R) \Rightarrow P \to R$

Rule 8（constructive reasoning）: $(P \to Q) \wedge (R \to S) \wedge (P \vee R) \Rightarrow Q \vee S$

Rule 9（destructive reasoning）: $(P \to Q) \wedge (R \to S) \wedge (\neg Q \vee \neg S) \Rightarrow \neg P \vee \neg R$

2. Understanding of Important Rules

The 9 rules mentioned above are the premise of the axiom system. With the help of supplementary rule 1 and set thought, we can find the cognitive rules of other rules.

1). Conjunction $\Rightarrow$ suppose there are two propositional formula A, B, if $A \to B$ is tautology, then it is called A tautological implication B, denoted as $A \Rightarrow B$.

2).Understanding of Rule 1: According to the definition of the conjunction, $P \Rightarrow P$ can be understood as $\neg P \vee P$, that is, tautology. Where the conjunction $\vee$ is understood as the union operation（$\cup$）in the set, and the tautology is understood as the complete set （U） in the set.

3).Understanding of Rule 2: Since $P \Rightarrow P$ is understood as $\neg P \vee P$ and is tautology, it can be understood by the set idea as $\complement_P \cup P = \mathrm{U}$, if the new set is calculated on the basis of the complete set U, it is still the complete set U, that is, the tautology. If $\complement_P \cup P \cup Q = \complement_P \cup (P \cup Q) = \mathrm{U}$, then $P \Rightarrow P \vee Q$.

4).Understanding of Rule 3: As above, the new set is still the complete set U, that is still U tautology, which uses the DeMorgan's theorem in the set. Therefore $\complement_P \cup P \cup \complement_Q = (\complement_P \cup \complement_Q) \cup P = (P \cap Q) \cup P = \mathrm{U}$,, then $P \wedge Q \Rightarrow P$.

5).Understanding of Rule 4: Combined with the understanding of 3) and 4) and on the basis of the complete set U, calculating and using DeMorgan's theorem, it is still the complete set U, that is, tautology. Therefore $U = (P \cap \complement_Q) \cup \complement_P \cup Q = \complement_{(\complement_P \cup Q)} \cup \complement_P \cup Q = \complement_{((\complement_P \cup Q) \cap P)} \cup Q$, then $(P \to Q) \wedge P \Rightarrow Q$.

6).Understanding of Rule 5: As is mentioned above, DeMorgan's theorem is used and calculated on the basis of the complete set U which is still the U, the tautology. Therefore $U = (P \cap \complement_Q) \cup Q \cup \complement_P = \complement_{(\complement_P \cup Q) \cap \complement_Q} \cup \complement_P$, then $(P \to Q) \wedge P \Rightarrow Q$.

7).Understanding of Rule 6: As is mentioned above, DeMorgan's theorem is used and calculated on the basis of the complete set U which is still the U, the tautology. Therefore $U = (\complement_P \cap \complement_Q) \cup P \cup Q = \complement_{P \cup Q} \cup P \cup Q$, then $(P \vee Q) \wedge \neg P \Rightarrow Q$.

8).Understanding of Rule 7: As is mentioned above, DeMorgan's theorem is used and calculated on the basis of the complete set U which is still the U, the tautology. Therefore $U=(P\cap\complement_Q)\cup(Q\cap\complement_R)\cup(\complement_P\cup R)=\complement_{\complement_P\cup Q}\cup\complement_{\complement_Q\cup R}\cup(\complement_P\cup R)$, then $(P\to Q)\wedge(Q\to R)\Rightarrow P\to R$.

9).Understanding of Rule 8：According to the understanding of 7), a new complete set is added to the U and calculated at the same time, the U is still the complete set U , which is the tautology. Therefore, $U=(P\cap\complement_Q)\cup(P\cap\complement_S)\cup(\complement_P\cap\complement_R)\cup(Q\cup S)$ $=\complement_{\complement_P\cup Q}\cup\complement_{\complement_R\cup S}\cup\complement_{P\cup R}\cup(Q\cup S)$ then $(P\to Q)\wedge(R\to S)\wedge(P\vee R)\Rightarrow Q\vee S$.

10).Understanding of Rule 9：According to the understanding of 9), a new complete set is added to the U and calculated at the same time, the U is still the complete set U , which is the tautology . Therefore $U=(P\cap\complement_Q)\cup(R\cap\complement_S)\cup(Q\cap S)\cup(\complement_P\cup\complement_R)$ $=\complement_{\complement_P\cup Q}\cup\complement_{\complement_R\cup S}\cup\complement_{\complement_Q\cup\complement_S}\cup\complement_{\complement_P\cup\complement_R}$, then $(P\to Q)\wedge(R\to S)\wedge(\neg Q\vee\neg S)\Rightarrow\neg P\vee\neg R$.

3. Conclusion

In propositional logic, the atomic proposition, as an indecomposable basic element, has great limitations in the reasoning process, so when reasoning with the reasoning theory in propositional calculus, we must master its important reasoning rules in order to master the ability of reasoning with the axiomatic system. 9 rules of reasoning combined with DeMorgan’s theorem in the set will transfer the abstract knowledge into familiar one, which has an excellent effect.

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